

HW IV , Math 530, Fall 2014

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QUESTION 1. (i) Let $(G, *)$ be a cyclic group of order 12, say $G = (a)$ for some $a \in G$. Then we know that G has unique subgroups of order 6, 4, 3, and 2. Construct each subgroup in terms of powers of a.

(ii) Let H be the subgroup of G of order 3 that you constructed in (i). Construct the Caley table for the group G/H.

(iii) Let $(G, *)$ be an infinite cyclic group, say $G = (a)$ for some $a \in G$. In terms of powers of a, construct all subgroups of G that contain the element a^{-7}. Construct all subgroups of G that contain the element a^6.

(iv) Let D, G be finite groups, and let $M = D \times G$. Let $(a, b) \in M$. Prove that $|(a, b)| = Lcm[|a|, |b|]$ (note that $Lcm[|a|, |b|] = |a||b|/gcd(|a|, |b|)$ is called the least common multiple of $|a|$ and $|b|$).

(v) Let $D = (Z_4, +) \times (Z_6, +)$, $G = (Z_9, +)$, and $M = D \times G$. Calculate the order of $(1, 2, 3) \in M$.

(vi) Let D be a finite cyclic group and G be an infinite cyclic group. Is $M = D \times G$ cyclic? if yes, then prove it. If no, then explain "no", i.e., do you mean sometimes yes and sometimes no or "no" means M is never cyclic.

(vii) Let D, G be infinite cyclic groups. Is $M = D \times G$ cyclic? if yes, then prove it. If no, then explain "no", i.e., do you mean sometimes yes and sometimes no or "no" means M is never cyclic.

(viii) Let D, G be finite cyclic groups, and $M = D \times G$. Prove that M is cyclic if and only if $gcd(|D|, |G|) = 1$. Is $(Z_8, +) \times (Z_{14}, +)$ cyclic?

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